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Simple Approach to Deriving Some Operator Ordering Formulas in Quantum Optics

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Abstract In this paper, we provide a simple and neat approach to some operators' normal ordering and antinormal ordering formulas in quantum optics. Namely, we directly adopt the generating function of Hermite polynomial and the Baker-Hausdorff formula, which differs from the existing ways. As an important byproduct, based on these operator identities, some useful mathematical integral formulas are easily given without really performing these integrations.

Keywords Hermite polynomial · Baker-Hausdorff formula · Operator ordering identities

1 Introduction

The operator ordering problem [1–3], which is one of the consequences of non-commutativity, plays an important role in the construction of quantum mechanical operators [4], phase space description of quantum mechanics [5], quantum c-number correspondence [6], etc. There are some definite operator orderings, such as normal ordering, antinormal ordering, and Weyl ordering. It is well known that normally ordered expansion of operators are very useful in handling miscellaneous calculations of expectation values of observables in the coherent states, because the matrix elements of the normally ordered operator function $F(a^\dagger, a)$ in coherent states $|z\rangle$ directly yields

$$F(z^*, z') = \langle z| :F(a^\dagger, a): |z'\rangle. \tag{1}$$

where the symbol $::$ stands for normal ordering, and a, $a^\dagger$ are the Bose creation, annihilation operators obeying $[a, a^\dagger] = 1$. The antinormal ordered form of $F(a^\dagger, a)$ is closely related

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to its diagonal coherent state representation. Therefore, it is an important topic to obtain the various ordered forms of the operators in quantum optics theory.

To our knowledge, there are three main means for handling operator ordering problems in most literatures, such as the Lie algebra method [7], Louisell's differential operation method via the coherent state representation [8], and the newly developed technique of integration within an ordered product (IWOP) of operators [9, 10]. Especially, the last one is a quite powerful method for arranging quantum mechanical operators into their ordered product form. In [11–18] using the IWOP technique, they have obtained the normal ordering forms of some operators. For example, Fan and Guo [15] derived the normally ordered form of the operator $H_m(fQ+gP)$ with $H_m(x)$ being the Hermite polynomial, i.e.,

$$H_m(fQ+gP)=(\sqrt{1-f^2-g^2})^m :H_m\left(\frac{fQ+gP}{\sqrt{1-f^2-g^2}}\right): \tag{2}$$

by using the integral formula of Hermite polynomial [19]

$$\int dx \exp\left[-\frac{(x-y)^2}{2u}\right]H_m(x)=\sqrt{2\pi u}(1-2u)^{m/2}H_m\left(\frac{y}{\sqrt{1-2u}}\right), \quad 2u\neq 1 \tag{3}$$

and the completeness relation of the eigenvector $|x\rangle_{f,g}$ of $fQ+gP$ as a pure Gaussian integration within the normal ordering form [17]

$$\int dx|x\rangle_{f,gg,f}\langle x|=\frac{1}{\sqrt{\pi(f^2+g^2)}}\int dx :\exp\left\{-\frac{[x-(fQ+gP)]^2}{f^2+g^2}\right\}:=1 \tag{4}$$

where x is the eigenvalue of $fQ+gP$, i.e. $(fQ+gP)|x\rangle_{f,g}=x|x\rangle_{f,g}$, Q and P are coordinate operator and momentum operator, respectively. However, from the above mentioned literatures including the above example, we notice that when the authors derive some operators' normal ordering forms by virtue of the IWOP technique, they must first know the completeness relation of the eigenvector of the operator to be ordered as a pure Gaussian integration within the normal ordering form and then make use of some complex mathematical integral formulas like (3). In the present work, in order to avoid these inconveniences, we adopt a simple approach to deriving some operators' normal ordering and antinormal ordering form such as $(\mu a+\nu a^\dagger)^m$, $H_m(\mu a+\nu a^\dagger)$, $a^n a^{\dagger m}$, and $a^{\dagger m}a^n$. Namely, we directly utilize the generating function of Hermite polynomial and the Baker-Hausdorff formula to deal with them. It is found that this way is much neater and easier than that of [15, 17, 18]. On the other hand, by virtue of the deduced operator ordering identities, we also easily obtain some mathematical integral formulas without really performing these integrations.

2 Normal Ordering and Antinormal Ordering of $(\mu a+\nu a^\dagger)^m$

To begin with, we derive the normal ordering form of $(\mu a+\nu a^\dagger)^m$ (For convenience, let $\Omega\equiv\mu a+\nu a^\dagger$). Recall that the generating function of single-variable Hermite polynomial $H_m(x)$ [20]

$$\sum_{m=0}^{\infty}\frac{t^m}{m!}H_m(x)=e^{2tx-t^2}, \quad H_m(x)=\frac{\partial^m}{\partial t^m}e^{2tx-t^2}\bigg|_{t=0}, \tag{5}$$

where

$$H_m(x)=\sum_{l=0}^{[m/2]}\frac{(-1)^l m!}{l!(m-2l)!}(2x)^{m-2l}, \tag{6}$$

and the Baker-Hausdorff formula [21]

$$e^{A+B}=e^A e^B e^{-\frac{1}{2}[A,B]}=e^B e^A e^{-\frac{1}{2}[B,A]} \tag{7}$$

with $[[A,B],A]=[[A,B],B]=0$, we easily obtain

$$\begin{aligned} e^{\lambda\Omega} &= :\exp\left(\lambda\Omega+\frac{1}{2}\lambda^2\mu\nu\right): \\ &= :\exp\left[2\left(-i\sqrt{\frac{\mu\nu}{2}}\lambda\right)\frac{i\Omega}{\sqrt{2\mu\nu}}-\left(-i\sqrt{\frac{\mu\nu}{2}}\lambda\right)^2\right]: \\ &= \sum_{m=0}^{\infty}\frac{(-i\sqrt{\frac{\mu\nu}{2}}\lambda)^m}{m!}:H_m\left(\frac{i\Omega}{\sqrt{2\mu\nu}}\right):, \end{aligned} \tag{8}$$

where we have used the commutation relation $[a,a^\dagger]=1$. Comparing (8) with the same power of λ in the expansion of $e^{\lambda\Omega}=\sum_{m=0}^{\infty}\frac{\lambda^m}{m!}\Omega^m$, we easily obtain the neat formula

$$\Omega^m=\left(-i\sqrt{\frac{\mu\nu}{2}}\right)^m:H_m\left(\frac{i\Omega}{\sqrt{2\mu\nu}}\right):, \tag{9}$$

i.e.

$$(\mu a+\nu a^\dagger)^m=\left(-i\sqrt{\frac{\mu\nu}{2}}\right)^m:H_m\left(i\frac{\mu a+\nu a^\dagger}{\sqrt{2\mu\nu}}\right):. \tag{10}$$

Further, by introducing $\mu=\frac{f-ig}{\sqrt{2}}$ and $\nu=\frac{f+ig}{\sqrt{2}}$, we can rewrite (10) as

$$(fQ+gP)^m=\left(\frac{\sqrt{f^2+g^2}}{2i}\right)^m:H_m\left(i\frac{fQ+gP}{\sqrt{f^2+g^2}}\right):, \tag{11}$$

where we have used $Q=\frac{a+a^\dagger}{\sqrt{2}}$ and $P=\frac{a-a^\dagger}{i\sqrt{2}}$. This result is also the same as (11) of [17]. Especially, when $f=1$ and $g=0$, we immediately obtain the normal ordering form of the operator X^m

$$Q^m=\left(-\frac{i}{2}\right)^m:H_m(iQ):, \tag{12}$$

and when $f=0$ and $g=1$, the normal ordering form of the operator P^m is also deduced

$$P^m=\left(-\frac{i}{2}\right)^m:H_m(iP):. \tag{13}$$

In [15, 17], with the help of the complex mathematical integral formula, they used the IWOP technique and the intermediate coordinate-momentum representation. Without doubt, our approach shows its features of simpleness and directness.

Similarly, we may derive the antinormal ordering form of $(\mu a+\nu a^\dagger)^m$ as well. With the help of (5) and (7), we also obtain that

$$\begin{aligned} e^{\lambda\Omega} &= \vdots e^{\lambda\Omega-\frac{1}{2}\lambda^2\mu\nu}\vdots \\ &= \sum_{m=0}^{\infty}\frac{(\sqrt{\frac{\mu\nu}{2}}\lambda)^m}{m!}\vdots H_m\left(\frac{\Omega}{\sqrt{2\mu\nu}}\right)\vdots \\ &= \sum_{m=0}^{\infty}\frac{\lambda^m}{m!}\Omega^m, \end{aligned} \tag{14}$$

where the symbol $\vdots\ \vdots$ represents the antinormal ordering. It follows from (14) that

$$\Omega^m = \left(\sqrt{\frac{\mu\nu}{2}}\right)^m \vdots H_m\left(\frac{\Omega}{\sqrt{2\mu\nu}}\right)\vdots, \tag{15}$$

i.e.

$$(\mu a+\nu a^\dagger)^m = \left(\sqrt{\frac{\mu\nu}{2}}\right)^m \vdots H_m\left(\frac{\mu a+\nu a^\dagger}{\sqrt{2\mu\nu}}\right)\vdots, \tag{16}$$

which is the antinormal ordering form of $(\mu a+\nu a^\dagger)^m$ or $(fQ+gP)^m$. Using (16), if letting $f=1$ and $g=0$, i.e. $\mu=\nu=\frac{1}{\sqrt{2}}$,

$$Q^m = \left(\frac{1}{2}\right)^m \vdots H_m(Q)\vdots \tag{17}$$

and when $f=0$ and $g=1$, i.e. $\mu=-\nu=\frac{1}{i\sqrt{2}}$,

$$P^m = \left(\frac{1}{2}\right)^m \vdots H_m(P)\vdots. \tag{18}$$

Here, compared with [17], our approach is simple and concise without using the antinormally ordered expansion's formula [22].

Moreover, in the same way we deduce the normal or antinormal ordering forms of the operator $H_m(\mu a+\nu a^\dagger)$. From (5) and (7) we see

$$\begin{aligned} \sum_{m=0}^{\infty}\frac{t^m}{m!}H_m(\mu a+\nu a^\dagger) &= \exp[2t(\mu a+\nu a^\dagger)-t^2] \\ &= :\exp\left[2\sqrt{1-2\mu\nu}t\frac{\mu a+\nu a^\dagger}{\sqrt{1-2\mu\nu}}-(\sqrt{1-2\mu\nu}t)^2\right]: \\ &= :\sum_{m=0}^{\infty}\frac{(\sqrt{1-2\mu\nu}t)^m}{m!}H_m\left(\frac{\mu a+\nu a^\dagger}{\sqrt{1-2\mu\nu}}\right):. \end{aligned} \tag{19}$$

Comparing the same power of t on the two sides of (19) leads to

$$H_m(\mu a+\nu a^\dagger) = (\sqrt{1-2\mu\nu})^m :H_m\left(\frac{\mu a+\nu a^\dagger}{\sqrt{1-2\mu\nu}}\right):. \tag{20}$$

When $\mu = \frac{f-ig}{\sqrt{2}}$ and $\nu = \frac{f+ig}{\sqrt{2}}$, (20) just recovers to (2) as expected. Note that in the second step of (19) we have supposed $1-2\mu\nu \neq 0$. For the case of $1-2\mu\nu = 0$,

$$\sum_{m=0}^{\infty}\frac{t^m}{m!}H_m(\mu a+\nu a^{\dagger}) = :\exp[2t(\mu a+\nu a^{\dagger})]: \\ = :\sum_{m=0}^{\infty}\frac{(2t)^m(\mu a+\nu a^{\dagger})^m}{m!}:. \tag{21}$$

It follows that

$$H_m(\mu a+\nu a^{\dagger}) = 2^m :(\mu a+\nu a^{\dagger})^m:, \tag{22}$$

whose special cases are

$$H_m(Q) = 2^m :Q^m:, \tag{23}$$

for $\mu = \nu = \frac{1}{\sqrt{2}}$, which is the same as (8) of [11], and

$$H_m(P) = 2^m :P^m:, \tag{24}$$

for $\mu = -\nu = \frac{1}{i\sqrt{2}}$.

In the similar way, $H_m(\mu a+\nu a^{\dagger})$'s antinormal ordering expansion is described as

$$\sum_{m=0}^{\infty}\frac{t^m}{m!}H_m(\mu a+\nu a^{\dagger}) = \vdots\exp\left[2\sqrt{1+2\mu\nu}\,t\frac{\mu a+\nu a^{\dagger}}{\sqrt{1+2\mu\nu}} - (\sqrt{1+2\mu\nu}\,t)^2\right]\vdots \\ = \vdots\sum_{m=0}^{\infty}\frac{(\sqrt{1+2\mu\nu}\,t)^m}{m!}H_m\left(\frac{\mu a+\nu a^{\dagger}}{\sqrt{1+2\mu\nu}}\right)\vdots, \tag{25}$$

then

$$H_m(\mu a+\nu a^{\dagger}) = (\sqrt{1+2\mu\nu})^m \vdots H_m\left(\frac{\mu a+\nu a^{\dagger}}{\sqrt{1+2\mu\nu}}\right)\vdots. \tag{26}$$

As $\mu = \frac{f-ig}{\sqrt{2}}$ and $\nu = \frac{f+ig}{\sqrt{2}}$, (26) turns into

$$H_m(fQ+gP) = (\sqrt{1+f^2+g^2})^m \vdots H_m\left(\frac{fQ+gP}{\sqrt{1+f^2+g^2}}\right)\vdots, \tag{27}$$

which seems a new operator identity. As its special cases, for $\mu = \nu = \frac{1}{\sqrt{2}}$,

$$H_m(X) = 2^{m/2}\vdots H_m\left(\frac{X}{\sqrt{2}}\right)\vdots, \tag{28}$$

and for $\mu = -\nu = \frac{1}{i\sqrt{2}}$

$$H_m(P) = 2^{m/2}\vdots H_m\left(\frac{P}{\sqrt{2}}\right)\vdots. \tag{29}$$

3 Normal Ordering Form of $a^n a^{\dagger m}$ and Antinormal Ordering Form of $a^{\dagger m} a^n$

In the following section, we directly derive the normal ordering form of $a^n a^{\dagger m}$ and the antinormal ordering form of $a^{\dagger m} a^n$ by virtue of the generation function of two-variable Hermite polynomial function and the Baker-Hausdorff formula.

From (7), it is easily obtained that

$$e^A e^B = e^B e^A e^{[A,B]}. \tag{30}$$

By introducing the two-variable Hermite polynomial function $H_{m,n}(\zeta,\xi)$ [23, 24]

$$H_{m,n}(\zeta,\xi) = \sum_{l=0}^{\min(m,n)} \frac{(-1)^l m!n!}{l!(m-l)!(n-l)!} \zeta^{m-l}\xi^{n-l}, \tag{31}$$

whose generating function is

$$\sum_{m,n=0}^{\infty} \frac{t^m \tau^n}{m!n!} H_{m,n}(\zeta,\xi) = \exp(-t\tau + t\zeta + \tau\xi), \tag{32}$$

or

$$H_{m,n}(\zeta,\xi) = \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \exp(-t\tau + t\zeta + \tau\xi)\bigg|_{t=t'=0}, \tag{33}$$

and using (30), we have

$$\begin{aligned} e^{\tau a} e^{t a^\dagger} &= {:}e^{\tau a + t a^\dagger + t\tau}{:} \\ &= {:}\exp[-i\tau(ia) + (-it) i a^\dagger - (-it)(-i\tau)]{:} \\ &= \sum_{m,n=0}^{\infty} \frac{(-it)^m(-i\tau)^n}{m!n!} {:}H_{m,n}(ia^\dagger, ia){:}. \end{aligned} \tag{34}$$

Then, comparing (34) with $e^{\tau a} e^{t a^\dagger} = \sum_{m,n=0}^{\infty} \frac{(\tau a)^n}{n!} \frac{(t a^\dagger)^m}{m!} = \sum_{m,n=0}^{\infty} \frac{\tau^n t^m}{n!m!} a^n a^{\dagger m}$ leads to

$$a^n a^{\dagger m} = (-i)^{m+n} {:}H_{m,n}(ia^\dagger, ia){:}, \tag{35}$$

which is a compact form of the normal ordering of $a^n a^{\dagger m}$. Similarly, the antinormal ordering form of $a^{\dagger m} a^n$ is described as

$$a^{\dagger m} a^n = \vdots H_{m,n}(a^\dagger, a) \vdots, \tag{36}$$

which can be deduced by comparing

$$\begin{aligned} e^{t a^\dagger} e^{\tau a} &= \vdots e^{\tau a + t a^\dagger - t\tau} \vdots = \vdots \exp[\tau a + t a^\dagger - t\tau] \vdots \\ &= \sum_{m,n=0}^{\infty} \frac{t^m \tau^n}{m!n!} \vdots H_{m,n}(a^\dagger, a) \vdots \end{aligned} \tag{37}$$

with $e^{t a^\dagger} e^{\tau a} = \sum_{m,n=0}^{\infty} \frac{t^m \tau^n}{n!m!} a^{\dagger m} a^n$. The results of (35) and (36) exactly agree with (15) and (26) of [18].

4 Applications

As some applications on the above operator identities, here we easily deduce some useful mathematic formulas. First of all, using (4) and (11), we may have

$$\begin{aligned}(fQ+gP)^m &= \int dx x^m |x\rangle_{f,gg,f}\langle x| \\ &= \frac{1}{\sqrt{\pi(f^2+g^2)}}\int dx x^m :\exp\left\{-\frac{[x-(fQ+gP)]^2}{f^2+g^2}\right\}: \\ &= \left(\frac{\sqrt{f^2+g^2}}{2i}\right)^m :H_m\left(\frac{i(fQ+gP)}{\sqrt{f^2+g^2}}\right):. \end{aligned} \tag{38}$$

By setting $fQ+gP\to y$ and $f^2+g^2\to\sigma$, the mathematical formula is given as

$$\int\frac{dx}{\sqrt{\pi}}x^m e^{-\frac{(x-y)^2}{\sigma}} = \frac{(\sqrt{\sigma})^{m+1}}{(2i)^m}H_m\left(\frac{iy}{\sqrt{\sigma}}\right). \tag{39}$$

Especially, when $\sigma=1$, (39) turns into

$$\int\frac{dx}{\sqrt{\pi}}x^m e^{-(x-y)^2} = (2i)^{-m}H_m(iy), \tag{40}$$

this result is in agreement with [19], without really performing this integration.

Moreover, considering (23) and (24), we obtain

$$H_m(Q)|0\rangle = 2^m :Q^m: |0\rangle = 2^{m/2}a^{\dagger m}|0\rangle = \sqrt{m!2^m}|m\rangle \tag{41}$$

and

$$H_m(P)|0\rangle = 2^m :P^m: |0\rangle = i^m 2^{m/2}a^{\dagger m}|0\rangle = i^m\sqrt{m!2^m}|m\rangle, \tag{42}$$

where we have used (1) as well as $a|0\rangle=0$, and $|n\rangle = a^{\dagger m}/\sqrt{m!}|0\rangle$ is number state in Fock space. It then follows from (41) and the coordinate eigenvector $|q\rangle = \frac{1}{\sqrt[4]{\pi}}\exp(-\frac{q^2}{2}+\sqrt{2}qa^\dagger-\frac{a^{\dagger 2}}{2})|0\rangle$ with $Q|q\rangle = q|q\rangle$ that

$$\langle q|m\rangle = \frac{1}{\sqrt{m!2^m}}\langle q|H_m(Q)|0\rangle = \frac{e^{-x^2/2}}{\sqrt{m!2^m\sqrt{\pi}}}H_m(q). \tag{43}$$

Similarly, using (42) and the momentum eigenvector $|p\rangle = \frac{1}{\sqrt[4]{\pi}}\exp(-\frac{p^2}{2}+\sqrt{2}ipa^\dagger+\frac{a^{\dagger 2}}{2})|0\rangle$ with $P|p\rangle = p|p\rangle$ leads to

$$\langle p|m\rangle = \frac{(-i)^m e^{-p^2/2}}{\sqrt{m!2^m\sqrt{\pi}}}H_m(p), \tag{44}$$

which agrees with the result derived via algebraic method in [25]. For some other applications on (23), one can see [26] in detail.

Next, using the overcompleteness relation of coherent state $\int\frac{d^2z}{\pi}|z\rangle\langle z| = 1$ and $a|z\rangle = z|z\rangle$, where

$$|z\rangle = \exp\left(-\frac{1}{2}|z|^2+za^\dagger\right)|0\rangle, \tag{45}$$

we see

$$\begin{aligned} a^n a^{\dagger m} &= \int \frac{d^2 z}{\pi} a^n |z\rangle\langle z| a^{\dagger m} \\ &= \int \frac{d^2 z}{\pi} z^n z^{*m} :\exp(-|z|^2 + z a^\dagger + z^* a - a^\dagger a):, \end{aligned} \tag{46}$$

where we have used $|0\rangle\langle 0| = :\exp(-a^\dagger a):$. It follows from (35) that

$$\int \frac{d^2 z}{\pi} z^n z^{*m} :\exp(-|z|^2 + z a^\dagger + z^* a - a^\dagger a): = (-i)^{m+n} :H_{m,n}(i a^\dagger, i a):. \tag{47}$$

Since within the normal ordering symbol a and $a^\dagger$ are commute, they may be treated as c-number. Setting $i a^\dagger \to \xi$ and $i a \to \eta$, we easily obtain

$$H_{m,n}(\xi,\eta) = (-1)^m e^{\xi\eta} \int \frac{d^2 z}{\pi} z^n z^{*m} \exp(-|z|^2 - \xi z + \eta z^*) \tag{48}$$

or

$$H_{m,n}(\xi,\eta) = (-1)^n e^{\xi\eta} \int \frac{d^2 z}{\pi} z^n z^{*m} \exp(-|z|^2 + \xi z - \eta z^*). \tag{49}$$

This integral formula is often used in the literatures [27–29].

On the other hand, recalling the P-representation in the coherent state $|z\rangle$ basis [8]

$$\rho = \int \frac{d^2 z}{\pi} P(z) |z\rangle\langle z|$$

and using (36) and (45), we can derive the following identity

$$\begin{aligned} \vdots H_{m,n}(a^\dagger, a) \vdots &= \int \frac{d^2 z}{\pi} H_{m,n}(z, z^*) |z\rangle\langle z| \\ &= \int \frac{d^2 z}{\pi} H_{m,n}(z, z^*) :\exp[-|z|^2 + z^* a + z a^\dagger - a a^\dagger]: \\ &= a^{\dagger m} a^n \end{aligned} \tag{50}$$

which implies a new integration formula by setting $a \to \eta$ and $a^\dagger \to \zeta^*$

$$\int \frac{d^2 z}{\pi} H_{m,n}(z, z^*) \exp[-(z^* - \zeta^*)(z - \eta)] = \zeta^{*m} \eta^n. \tag{51}$$

Finally, Using $[a^\dagger, a^n] = -n a^{n-1}$ and $[a, a^{\dagger m}] = m a^{\dagger m-1}$, we have

$$a^{\dagger m+1} a^n = a^{\dagger m} a^n a^\dagger - n a^{\dagger m} a^{n-1} \tag{52}$$

and

$$a^{\dagger m} a^{n+1} = a a^{\dagger m} a^n - m a^{\dagger m-1} a^n. \tag{53}$$

Then considering (36), it is obtained from (52) and (53) that

$$\vdots H_{m+1,n}(a^\dagger, a) \vdots = \vdots H_{m,n}(a^\dagger, a) \vdots a^\dagger - n \vdots H_{m,n-1}(a^\dagger, a) \vdots \tag{54}$$

and

$$\vdots H_{m,n+1}(a^\dagger, a)\vdots = a \vdots H_{m,n}(a^\dagger, a)\vdots - m \vdots H_{m-1,n}(a^\dagger, a)\vdots \tag{55}$$

which indicate the recurrence relations of $H_{m,n}(\xi, \eta)$,

$$H_{m+1,n}(\xi, \eta) - \xi H_{m,n}(\xi, \eta) + n H_{m,n-1}(\xi, \eta) = 0 \tag{56}$$

and

$$H_{m,n+1}(\xi, \eta) - \eta H_{m,n}(\xi, \eta) + m H_{m-1,n}(\xi, \eta) = 0. \tag{57}$$

So it is seen that (35) and (36) also provides an approach to deriving the well-known recurrence relations two-variable Hermite polynomial.

5 Conclusions

In summary, we directly utilize the generating function of Hermite polynomial and the Baker-Hausdorff formula to some operators' normal ordering and antinormal ordering form such as $(\mu a + \nu a^\dagger)^m$, $H_m(\mu a + \nu a^\dagger)$, $a^n a^{\dagger m}$, and $a^{\dagger m} a^n$. Compared with the existing ways [15, 17, 18], our method is a simple and neat for deriving these operator identities. As a important byproduct, based on these operator identities, some useful mathematical integral formulas are easily given without really performing these integrations.

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